Worcester County Mathematics League

WOCOMAL Varsity Meet #1

Coaches' Booklet

October 11, 2006

Round 1:   **Arithmetic (NO CALCULATORS)**

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1.   Simplify: $16 \div 2[8 - 3(4 - 2)] + 1$.

2.   If $(x,y) = (4.8, B)$ gives the coordinates of a point on the graph of $y = \dfrac{5}{8}x + 5$, then what is the value of B?

3.   Suppose the operation $\otimes$ is defined on the set of integers by $a \otimes b = a + 2b$. Also, for any integer b, the value of $1492 \otimes (b \otimes 1492)$ is the same as $x \otimes b$. What is the value of x?

ANSWERS

(1 pt.)   1. _____

(2 pts.)   2. _____

(3 pts.)   3. _____

Auburn, Bancroft

Round 2:   Algebra 1 (open)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1.   If x and y are natural numbers, then how many solutions does the equation
        $x + y = 12$ have?

2.   Susan left her home, on foot, at 9:00 am at 1.5 mph. At 10:00 am her friend,
        following the same route, ran at 5.25 mph to meet her. How many minutes, after
        10:00 am, did the two friends meet?

3.   What is the sum of the squares of the roots of $x^4 - 5x^2 + 6 = 0$?

ANSWERS

(1 pt.)   1. _____

(2 pts.)   2. _____

(3 pts.)   3. _____

Clinton, Barttlett, Bromfield

Round 3:   Set Theory

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1.   If $A = \{0,1,2,3,4,5,6,7\}$, set A has how many non-empty, proper subsets?

2.   If $n \in \{Integers\}\, and\, \{2n-3 \geq 23\} \cap \left\{\dfrac{-2}{3}(n-10) \geq -2\right\} \neq \varnothing$, then what is the value of n?

3.   How many subsets of $\{1,2,3,4,5,6,7,8\}$ contain at least one even integer?

ANSWERS

(1 pt.)   1. _____

(2 pts.)   2. _____

(3 pts.)   3. _____

Leicester, West Boylston, Worcester Academy

Round 4:   Measurement

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1.   The sides of a triangle are consecutive odd integers. Its perimeter is 81 cm.
        Determine the length of the shortest side in cm.

2.   An isosceles triangle has a 10-inch base and two 13-inch sides. What other value,
        in inches, can the base have and still produce a triangle with the same area?

3.   Determine the volume of the cube below if the area of $\triangle ABC$ is $16\sqrt{3}\ cm^2$.
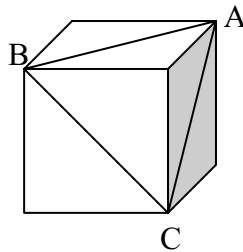
ANSWERS

(1 pt.)   1. _____ cm

(2 pts.)   2. _____ in

(3 pts.)   3. _____ cm^3

Tahanto, St. John's, Worcester Academy

Round 5:   Polynomial Equations (NO CALCULATORS)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1.   Determine the missing term for this quadratic equation so that it will have only
        one solution.

$$9x^2 - 24x + ? = 0$$

2.   One root of $mx^2 - 10x + 3 = 0$ is $\dfrac{2}{3}$ of the other root. What is the sum of the roots?

3.   The equation $2x^4 - 3x^3 - 14x^2 - 22x - 8 = 0$ has two real and two complex
        solutions. What is the product of the two complex solutions?

ANSWERS

(1 pt.)   1. _____

(2 pts.)   2. _____

(3 pts.)   3. _____

Shrewsbury, Hudson, Worcester Academy

TEAM ROUND

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM AND ON THE
SEPARATE TEAM ANSWER SHEET.   (2 points each)

1.   The real numbers x and y satisfy the equations $y(2x-3y)=19$ and
        $x(3x-4y)=94$. Evaluate $|x-y|$.

2.   If an equilateral triangle and a square have the same perimeters and the triangle's
        area is $\dfrac{64\sqrt{3}}{9}$ units^2, what is the area of the square?

3.   If $A=\{primes>0\}, B=\{factors\ldots of\,66\}, and\,C=\{Natural\ldots Numbers\geq 6\}$,
        then $A\cap B\cap C=?$

4.   If $xy=x+y$ where x and y are integers, then how many solutions does the
        equation have, where a solution is an ordered pair (x,y)?

5.   Write a quadratic polynomial having as its two zeros the numbers obtained by
        increasing each root of $x^2-2x-5=0$ by the reciprocal of the other. Write your
        answer in the form x^2 + bx + c where b,c $\in \{real\ldots numbers\}$.

6.   If $ac+bd=19$ and $ad-bc=94$, determine the value of $(a^2+b^2)(c^2+d^2)$.

7.   Suppose the operation $\oplus$ is defined on the set of integers by $a\oplus b=a+3b$.
        For what integer value of k does the equation $a\oplus(b\oplus a)=(ka)\oplus b$ hold?

8.   Two numbers are such that their difference, their sum, and their product are to
        each other as 1:7:24. Their product must equal what number?

9.   Suppose that the numbers 1,2,4,8,16,32,64,128,256 are placed into the table
        below in such a way that the product of the numbers appearing in any row,
        column or diagonal is the same. What is the value of this common product?

Bromfield, Notre Dame, Tahanto, Bartlett, St. John's, Worcester Academy

Round 1: Arithmetic

1. (1 pt.)　　　　　5

2. (2 pts.)　　　　8

3. (3 pts.)　　　　7460

Round 2: Algebra-open

1.　　(1 pt.)　　　11

2.　　(2 pts.)　　　24

3.　　(3 pts.)　　　10

Round 3: Set Theory

1.　　(1 pt.)　　　254

2.　　(2 pts.)　　　13

3.　　(3 pts.)　　　240

Round 4: Measurement

1.　　(1 pt.)　　　25

2.　　(2 pts.)　　　24

3.　　(3 pts.)　　　$128\sqrt{2}$

Round 5: Polynomial Equations

1.　　(1 pt.)　　　16

2.　　(2 pts.)　　　$\dfrac{5}{4}$

3.　　(3 pts.)　　　2

TEAM ROUND (2 pts. Each)

1.   5

2.   16

3. $\{11\}$

4.   2

5. $x^2 - \dfrac{8}{5}x - \dfrac{16}{5}$

6.   9197

7.   10

8.   48

9.   4096

TEAM ROUND

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM AND ON THIS SEPARATE TEAM ANSWER SHEET.   (2 points each)

1.   _____

2.   _____ **units**^{2}

3.   _____

4.   _____

5.   _____

6.   _____

7.   _____

8.   _____

9.   _____

October 11, 2006   **WOCOMAL Varsity Meet**

TEAM ROUND   **School: _____**

Team #:_____

Team Members:

    1.   _____

    2.   _____

    3.   _____

    4.   _____

    5.   _____

Total Points for Team Round: _____